Q.1	Evaluate : $\int \frac{a^x}{\sqrt{1-a^{2x}}}dx.$
Q.2	Evaluate $\int \frac{\sec x}{1+\cos ecx}dx$
Q.3	Evaluate: $\int e^x \frac{x}{(x+1)^2}dx.$
Q.4	Evaluate: $\int \frac{\sin^3 x+\cos^3 x}{\sin^2 x\cos^2 x}dx$.
Q.5	Evaluate: $\int \frac{\cos 2x-\cos 2\alpha}{\cos x-\cos\alpha}dx$.
Q.6	Evaluate: $\int \frac{dx}{e^x+e^{-x}}$.
Q.7	Evaluate: $\int \{1+2\tan x(\tan x+\sec x)\}^{1/2}dx$.
Q.8	Evaluate : $\int \frac{1}{(\sin x+4)(\sin x-1)}dx$.
Q.9	Evaluate: $\int \sqrt{\frac{1+x}{x}}dx.$
Q.10	Evaluate: $\int \frac{\cos x-\sin x}{1+\sin 2x}dx$.
Q.11	Evaluate : $\int (x^4+x^2+1)d(x^2)$.
Q.12	Evaluate : $\int \frac{1}{x^2(x^4+1)^{3/4}}dx$
Q.13	Evaluate: $\int \frac{\sin^{-1}x}{(1-x^2)^{3/2}}dx.$
Q.14	Evaluate : $\int (x+1)\sqrt{1-x-x^2}dx$.
Q.15	Evaluate: $\int \frac{3\cos x+2}{\sin x+2\cos x+3}dx.$.
Q.16	Evaluate $\int \sqrt{\frac{\sin(x-\alpha)}{\sin(x+\alpha)}}dx.$
Q.17	Evaluate: $\int \frac{\sqrt{\cos 2x}}{\sin x}dx$.
Q.18	Evaluate: $\int \frac{dx}{(x-1)\sqrt{2x+3}}$.
Q.19	Evaluate: $\int \log(1+x^2)dx.$
Q.20	Evaluate $\int (x-2)\sqrt{2x^2-6x+5}dx.$
Q.21	Evaluate: $\int \frac{(3\sin x-2)\cos x}{5-\cos^2 x-4\sin x}dx.$.
Q.22	Evaluate: $\int \frac{e^{\tan^{-1}x}}{(1+x^2)^2}$ **dx .**
Q.23	Evaluate: $\int \cos 2\theta \log\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)d\theta$.
Q.24	Evaluate: $\int \frac{dx}{(\sin x+\sin 2x)}$ .sol:
Q.25	Evaluate : $\int \frac{\tan x+\tan^3 x}{1+\tan^3 x}dx$.
Q.26	Evaluate: $\int \sin^{-1}\sqrt{\frac{x}{a+x}}dx$.
Q.27	Evaluate: $\int \frac{dx}{\cos 2x+3\sin^2 x}$

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Q.28	Evaluate: $\int \frac{x^3+x^2+2x+1}{x^2-x+1}dx.$
Q.29	Evaluate: $\int \frac{\sin x}{\sin 4x}dx$.
Q.30	Evaluate: $\int \frac{x^2}{x^2+7x+10}dx.$
Q.31	Evaluate: $\int (\log x)^2 dx.$
Q.32	Evaluate: $\int \sin(e^x)d(e^x)$..
Q.33	Evaluate: $\int \frac{4\sin x+5\cos x}{5\sin x+4\cos x}dx.$
Q.34	Evaluate: $\int \frac{dx}{(x^2+4)\sqrt{x^2-4}}$.
Q.35	Evaluate: $\int \frac{1+\sin x}{\sin x(1+\cos x)}dx.$
Q.36	Evaluate : $\int \frac{2\sin 2\phi-\cos\phi}{6-\cos^2\phi-4\sin\phi}d\phi.$
Q.37	Evaluate: $\int e^x\left(\frac{2+\sin 2x}{1+\cos 2x}\right)dx.$
Q.38	Evaluate : $\int \frac{\sin x+\cos x}{9+16\sin 2x}dx$.
Q.39	Evaluate : $\int \frac{1}{\sin x-\sin 2x}dx$.
Q.40	Evaluate: $\int \frac{x^2+1}{x^4-x^2+1}dx$.
Q.41	Evaluate: $\int \frac{x^2}{(x\sin x+\cos x)^2}dx$.
Q.42	Evaluate : $\int e^x Sin^2 4x dx$.
Q.43	Evaluate : $\int (7x-2)\sqrt{3x+2}dx.$
Q.44	Evaluate: $\int \frac{\log x}{x^2}dx.$
Q.45	Evaluate: $\int \frac{dx}{(x^2+3)\sqrt{x-1}}$.
Q.46	Evaluate : $\int \frac{8x+13}{\sqrt{4x+7}}dx.$
Q.47	Evaluate : $\int e^{2x}\left(\frac{\sin 4x-2}{1-\cos 4x}\right)dx$.
Q.48	Evaluate : $\int \frac{\sqrt{x^2+1}[\log(x^2+1)-2\log x]}{x^4}dx$.
Q.49	Evaluate : $\int \frac{\cos 2x}{(\cos x+\sin x)^2}dx.\backslash$
Q.50	Evaluate : $\int \sin x\sin 2x\sin 3x dx.$
Q.51	**Evaluate** $\int \frac{x^2+x+1}{(x-1)^3}dx.$
Q.52	Evaluate : $\int [1+2\cot x(\cot x+\cos ecx)]^{1/2}dx.$
Q.53	Evaluate: $\int \frac{x^2+5x+3}{x^2+3x+2}dx.$
Q.54	Evaluate: $\int \frac{1}{4+3\tan x}dx.$
Q.55	Write a value of $\int e^{3\log x}(x^4)dx.$
Q.56	Evaluate : $\int \frac{x+1}{\sqrt{2x+1}}dx$.

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Q.57	Evaluate: $\int (\tan x-\cot x)^2\ dx$.
Q.58	Evaluate: $\int \frac{\sin^{-1} x}{x^2}dx.$
Q.59	Evaluate: $\int \frac{x^2 \sin^{-1} x}{\left(1-x^2\right)^{3/2}}dx.$
Q.60	Evaluate: $\int \frac{dx}{\sqrt{5-4e^x-e^{2x}}}$.
Q.61	Evaluate: $\int \frac{1}{\sin x+\sqrt{3}\cos x}dx.$
Q.62	Evaluate: $\int \frac{1}{5+7\cos x+\sin x}dx.$
Q.63	Evaluate : $\int \frac{1}{3x^2+13x-10}dx.$
Q.64	Evaluate: $\int \frac{\sin 2x}{(a+b\cos x)^2}dx.$
Q.65	Evaluate: $\int \frac{a\ dx}{b+ce^x}$
Q.66	Evaluate : $\int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)}dx$.
Q.67	Evaluate : $\int \frac{a^x}{\sqrt{1-a^{2x}}}dx.$
Q.68	Evaluate : $\int \frac{1}{x(x^n+1)}dx.$
Q.69	Evaluate $\int \frac{x^2}{(x-1)^3(x+1)}dx.$
Q.70	Evaluate $\int \frac{x^3}{(x-1)(x-2)(x-3)}dx.$
Q.71	Evaluate: $\int x\log(1+x)dx.$
Q.72	Evaluate: $\int \sec^3 x\ dx$.
Q.73	Evaluate: $\int \frac{2x+3}{(x-1)(x^2+1)}dx$.
Q.74	Evaluate: $\int\left\{\log(\log x)+\frac{1}{(\log x)^2}\right\}dx$.
Q.75	Evaluate : $\int \frac{dx}{\sqrt{x}+\sqrt{x-2}}$.
Q.76	Evaluate : $\int \frac{\log x}{(1+\log x)^2}dx$.
Q.77	Evaluate : $\int (\sin^{-1} x)^2\, dx$.
Q.78	Evaluate: $\int \frac{1}{\sin x+\sqrt{3}\cos x}dx.$
Q.79	Evaluate: $\int \frac{1}{\sqrt{Cos^3 x Cos(x+\alpha)}}dx$.
Q.80	Evaluate: $\int \frac{x^3+x}{x^4-9}dx.$
Q.81	Evaluate: $\int \frac{1}{2e^{2x}+3e^x+1}dx.$
Q.82	Evaluate: $\int \frac{1}{(2\sin x+3\cos x)^2}dx.$

Q.83	Evaluate: $\int \frac{\sin x\cos x}{a\cos^2 x+b\sin^2 x}dx$.
Q.84	Evaluate: $\int \frac{e^x(x+1)}{\cos^2(xe^x)}dx$.
Q.85	Evaluate: $\int \frac{\operatorname{cosec} x}{\log \tan \frac{x}{2}}dx$ ·
Q.86	Evaluate : $\int \frac{dx}{x[(\log x)^2+4\log x-1]}$.
Q.87	Evaluate: $\int \frac{\cos x}{\sqrt{\sin^2 x-2\sin x-3}}dx.$
Q.88	Evaluate: $\int \frac{1}{(\sin x-2\cos x)(2\sin x+\cos x)}dx.$
Q.89	Evaluate: $\int \frac{1}{\sqrt{1-e^{2x}}}dx.$
Q.90	Evaluate : $\int \sqrt{\sec x-1}dx.$
Q.91	Evaluate: $\int \frac{1}{2-3\cos 2x}dx.$
Q.92	Evaluate: $\int \frac{dx}{1+x+x^2+x^3}$.
Q.93	Evaluate: $\int \frac{1-\cos x}{\cos x(1+\cos x)}dx$ ·
Q.94	Evaluate : $\int e^x \frac{(x^2+1)}{(x+1)^2}dx$ ·
Q.95	Evaluate : $\int \sqrt{4x^2+9}\,dx$.
Q.96	Evaluate: $\int \frac{\sin x}{\sin 3x}dx.$
Q.97	Evaluate : $\int \frac{dx}{16+9e^{-2x}}$.
Q.98	Evaluate : $\int \frac{1}{x\{6(\log x)^2+7\log x+2\}}dx$.
Q.99	Evaluate : $\int \frac{1}{3+4\tan x}dx$.
Q.100	Evaluate: $\int e^{-2x}\sin 3x\,dx$.
Q.101	Evaluate: $\int \sqrt{\frac{x}{a^3-x^3}}dx$ ·
Q.102	Evaluate: $\int \sqrt{e^x-1}dx$.
Q.103	Evaluate: $\int x^3 \log 2x dx.$
Q.104	Evaluate: $\int x\sin^{-1} x dx.$
Q.105	Evaluate : $\int \frac{e^{5\log_e x}-e^{4\log_e x}}{e^{3\log_e x}-e^{2\log_e x}}dx$.
Q.106	Evaluate: $\int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx.$
Q.107	Evaluate : $\int \tan x\tan 2x\tan 3x\,dx$.
Q.108	Evaluate: $\int \frac{x\tan^{-1} x}{(1+x^2)^{3/2}}dx$.
Q.109	Evaluate : $\int \frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}dx$.
Q.110	Evaluate $\int \sqrt{4x^2+9}\,dx$.

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Q.111	Evaluate : $\int \frac{dx}{\sin^2 x \cos^2 x}$.
Q.112	Evaluate : $\int \frac{e^{-x}}{16+9e^{-2x}} dx$.
Q.113	Evaluate : $\int \frac{dx}{x \log x \log(\log x)}$.
Q.114	Evaluate : $\int \frac{\sin 5x}{\sin 2x \sin 3x} dx$.
Q.115	Evaluate : $\int 2^{2^{2^x}} 2^{2^x} 2^x dx$.
Q.116	Evaluate : $\int x^2 \frac{\tan^{-1} x^3}{1+x^6} dx$.
Q.117	Evaluate : $\int \frac{\sin 2x}{1+\cos^2 x} dx$.
Q.118	Evaluate: $\int \sec^p x \tan x \, dx$.
Q.119	Evaluate : $\int \frac{1}{4+5\cos x} dx$.
Q.120	Evaluate : $\int \frac{1}{2e^{2x}+3e^x+1} dx$.
Q.121	Evaluate : $\int \sqrt{\frac{a+x}{a-x}} dx$.
Q.122	Evaluate: $\int \tan^4 x \, dx$.
Q.123	Evaluate : $\int \frac{\sin^6 x+\cos^6 x}{\sin^2 x \cos^2 x} dx$.
Q.124	Evaluate: $\int \frac{\sin 2x}{\sin^4 x+\cos^4 x} dx$.
Q.125	Evaluate: $\int \frac{(x^4-x)^{1/4}}{x^5} dx$.
Q.126	Evaluate: $\int \frac{dx}{\sin(x-a)\sin(x-b)}$.
Q.127	Evaluate: $\int \frac{\sqrt{(x^2-a^2)}}{x} dx$.
Q.128	*Evaluate :* $\int \frac{1}{x^2(x^4+1)^{3/4}} dx$.
Q.129	Evaluate $\int \frac{x^3}{\sqrt{x^2-1}} dx$.
Q.130	Evaluate: $\int \frac{dx}{x(x^n+1)}$.
Q.131	Evaluate : $\int \frac{3\cos x+2\sin x}{4\cos x+3\sin x} dx$.
Q.132	*Evaluate:* $\int \frac{dx}{(x+1)^{1/3}+(x+1)^{1/2}}$.
Q.133	Evaluate: $\int \frac{2\tan x+3}{3\tan x+4} dx$.
Q.134	Evaluate: $\int \frac{8\cot x+1}{3\cot x+2} dx$.
Q.135	Evaluate: $\int \frac{x^2}{x^4+x^2+16} dx$.

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Q.136	Evaluate : $\int \sqrt{\tan x} dx$.
Q.137	Evaluate : $\int \frac{2\sin x-3\cos x+1}{3\sin x+4\cos x+5} dx$.
Q.138	Evaluate : $\int \cos 2x \cos 4x \cos 6x dx$.
Q.139	Evaluate : $\int \sqrt{\left(\frac{1-\sqrt{x}}{1+\sqrt{x}}\right)} dx$.
Q.140	Evaluate : $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$.
Q.141	Evaluate: $\int \frac{dx}{\sqrt{\sin^3 x \sin(x+\alpha)}}$.
Q.142	Evaluate : $\int \sqrt{\frac{\sin(x-\alpha)}{\sin(x+\alpha)}} dx$.
Q.143	Evaluate : $\int \sqrt{\frac{\cos x - \cos^3 x}{1-\cos^3 x}} dx$.
Q.144	Evaluate : $\int (x+1)\sqrt{1-x-x^2} dx$.
Q.145	Evaluate : $\int \frac{2\sin 2x-\cos x}{6-\cos^2 x-4\sin x} dx$.
Q.146	Evaluate : $\int \frac{\sin x+\cos x}{\sqrt{\sin x.\cos x}} dx$.
Q.147	Evaluate: $\int \frac{6x-5}{\sqrt{3x^2-5x+1}} dx$.
Q.148	Evaluate: $\int \frac{x\sin^{-1} x}{\sqrt{1-x^2}} dx$.
Q.149	Evaluate $(3x-2)\sqrt{x^2+x+1} dx$.
Q.150	Evaluate: $\int \left(\frac{1}{\log x}-\frac{1}{(\log x)^2}\right) dx$.
Q.151	Evaluate $\int \sqrt{7x-10-x^2} dx$.
Q.152	Evaluate $\int e^{2x} \sin x \cos x dx$.
Q.153	Evaluate: $\int \frac{e^x(x-3)}{(x-1)^3} dx$.
Q.154	Evaluate: $\int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$.
Q.155	Evaluate: $\int \frac{x}{\sqrt{8+x-x^2}} dx$.
Q.156	Evaluate: $\int \sin^{-1}\sqrt{\frac{x}{a+x}} dx$.
Q.157	Evaluate: $\int e^x \left(\frac{1-\sin x}{1+\cos x}\right) dx$.
Q.158	Evaluate : $\int \frac{\sin x}{(1+\cos x)^2} dx$.
Q.159	Evaluate : $\int \frac{\tan x}{\sqrt{\cos x}} dx$.
Q.160	Evaluate: $\int e^{2x} \left(\frac{1+\sin 2x}{1+\cos 2x}\right) dx$.
Q.161	Evaluate $\int \frac{2x-1}{(x-1)(x+2)(x-3)}$

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Q.162	Evaluate $\int \frac{x^2+1}{(x^2+2)(2x^2+1)}dx.$
Q.163	Evaluate $\int \frac{x^2}{(x^2+1)(x^2+4)}dx.$
Q.164	Evaluate $\int \frac{x}{(x-1)(x^2+4)}dx.$
Q.165	Evaluate $\int \frac{8}{(x+2)(x^2+4)}dx.$
Q.166	Evaluate $\int \frac{x^2}{(x-1)^3(x+1)}dx.$
Q.167	Evaluate $\int \frac{3x^2+2x+1}{(x-1)^3}dx.$
Q.168	Evaluate $\int \frac{x^2+1}{(x-1)^2(x+3)}dx.$
Q.169	Evaluate $\int \frac{1}{(x+1)\sqrt{x^2-1}}dx.$
Q.170	Evaluate $\int \frac{1}{(x^2-4)\sqrt{x+1}}dx.$
Q.171	Evaluate $\int \frac{\cos\theta}{(2+\sin\theta)(3+4\sin\theta)}d\theta.$
Q.172	Evaluate $\int \frac{2x}{(x^2+1)(x^2+2)}dx.$
Q.173	Evaluate $\int \frac{1}{(x-1)\sqrt{x^2+4}}dx.$
Q.174	Evaluate $\int \frac{3x+1}{(x-2)^2(x+2)}dx.$
Q.175	Evaluate $\int \frac{\sqrt{x^2+1}}{x^4}dx.$
Q.176	Evaluate $\int \frac{1}{\sec x+\operatorname{cosec} x}dx.$
Q.177	Evaluate $\int \frac{x^2+x+1}{(x-1)^3}dx.$
Q.178	Evaluate $\int \frac{x^2-1}{x^4+x^2+1}dx.$
Q.179	Evaluate $\int \frac{1}{\sin^4 x+\cos^4 x}dx.$
Q.180	Evaluate $\int \left\{\sqrt{\tan\theta}+\sqrt{\cot\theta}\right\}d\theta.$
Q.181	Evaluate $\int \frac{1}{x^4+5x^2+16}dx.$
Q.182	Evaluate $\int \frac{x^3}{(x-1)(x-2)(x-3)}dx.$
Q.183	Evaluate $\int \sqrt{\operatorname{cosec} x-1}\,dx.$
Q.184	Evaluate $\int \frac{1}{x(x^5+1)}dx.$
Q.185	Evaluate $\int \frac{\tan^2 x\sec^2 x}{1+\tan^6 x}dx$
Q.186	Evaluate $\int \frac{1}{\sin x+\sin 2x}dx.$
Q.187	Evaluate $\int \frac{\cos^5 x}{\sin x}dx.$

Q.188	Evaluate $\int \sqrt{\frac{a+x}{x}}dx$
Q.189	Evaluate $\int \frac{1}{\sec x+\sin x}dx.$
Q.190	Evaluate $\int \frac{x^2}{x^4-3x^2+25}dx.$
Q.191	Evaluate $\int \frac{x^2-3x+1}{x^4+x^2+1}dx.$
Q.192	Evaluate $\int \sqrt{\frac{x+3}{x+2}}dx.$
Q.193	Evaluate $\int \frac{\sqrt{\cos 2x}}{\cos x}$ **dx .**
Q.194	Evaluate $\int \frac{\sin x}{\sin^3 x+\cos^3 x}dx.$
Q.195	Evaluate $\int \frac{(x+1)}{x(1+xe^x)}dx.$
Q.196	Evaluate $\int \frac{\sqrt{1-\sin x}}{1+\cos x}e^{-x/2}dx.$
Q.197	Evaluate $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}dx.$
Q.198	**Evaluate:** $\int \frac{1+\cot x}{x+\log\sin x}dx$
Q.199	Evaluate $\int \frac{\cos x}{\sin^3 x+\cos^3 x}dx.$
Q.200	Evaluate $\int \tan^{-1}\sqrt{x}dx$
Q.201	**Evaluate:** $\int \frac{\tan^{-1} x}{(1+x)^2}$ **dx .**
Q.202	**Evaluate:** $\int \frac{\log x}{(1+x)^3}$ **dx .**
Q.203	Evaluate $\int \cos^{-1}(1/x)dx.$
Q.204	Evaluate $\int x^2e^{x^3}\cos\left(e^{x^3}\right)dx.$
Q.205	Evaluate $\int \frac{\sin 2x}{a^2\sin^2 x+b^2\cos^2 x}dx.$
Q.206	**Evaluate:** $\int \frac{\sqrt{1+x^2}}{x}dx.$
Q.207	**Evaluate :** $\int \frac{2+\sin x}{1+\cos x}e^{x/2}dx$.
Q.208	Evaluate : $\int \frac{\sqrt{\cos x}}{\sin x}$ **dx .**
Q.209	**Evaluate :** $\int \frac{dx}{\cos(x-a)\cos(x-b)}$.
Q.210	**Evaluate:** $\int \frac{dx}{1-\sin^4 x}$.
Q.211	Evaluate : $\int \frac{\cot\theta+\cot^3\theta}{1+\cot^3\theta}dx.$
Q.212	The anti derivative of $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$.
Q.213	If $\frac{d}{dx}f(x)=4x^3-\frac{3}{x^4}$ such that $f(2)=0$. Then $f(x)$

Q.214	$\int \frac{10x^9 + 10^x \log_{e^{10}} dx}{x^{10} + 10^x}$
Q.215	Find $\int \frac{x^2+1}{x^2-5x+6} dx$
Q.216	Find $\int \frac{3x-2}{(x+1)^2(x+3)} dx$
Q.217	Find $\int \frac{(3\sin\phi - 2)\cos\phi}{5-\cos^2\phi - 4\sin\phi} d\phi$
Q.218	Find $\int \frac{x^2+x+1\, dx}{(x+2)(x^2+1)}$
Q.219	Find $\int \frac{x^4\, dx}{(x-1)(x^2+1)}$
Q.220	Integrate $\frac{\sin^8 - \cos^8 x}{1-2\sin^2 x \cos^2 x}$
	**********//*************

FUNDAMENTAL INTEGRATION FORMULAS

(i) $\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n, \Rightarrow \int x^n dx = \frac{x^{n+1}}{n+1} + C$

(ii) $\frac{d}{dx}(\log x) = \frac{1}{x} \Rightarrow \int \frac{1}{x} dx = \log|x| + C$

(iii) $\frac{d}{dx}(e^x) = e^x \Rightarrow \int e^x dx = e^x + C$

(iv) $\frac{d}{dx}\left(\frac{a^x}{\log_e a}\right) = a^x, \Rightarrow \int a^x dx = \frac{a^x}{\log a} + C$

(v) $\frac{d}{dx}(-\cos x) = \sin x \Rightarrow \int \sin x dx = -\cos x + C$

(vi) $\frac{d}{dx}(\sin x) = \cos x \Rightarrow \int \cos x dx = \sin x + C$

(vii) $\frac{d}{dx}(\tan x) = \sec^2 x \Rightarrow \int \sec^2 x dx = \tan x + C$

(viii) $\frac{d}{dx}(-\cot x) = \cos ec^2 x$

$\Rightarrow \int \cos ec^2 x dx = -\cot x$

(ix) $\frac{d}{dx}(\sec x) = \sec x \tan x$

$\Rightarrow \int \sec x \tan x\, dx = \sec x$

(x) $\frac{d}{dx}(-\cos ec x) = \cos ec x \cot x \Rightarrow$

$\int \cos ec x \cot x\, dx = -\cos ec x + C$

(xi) $\frac{d}{dx}(\log \sin x) = \cot x$

$\Rightarrow \int \cot x\, dx = \log|\sin x|$

(xii) $\frac{d}{dx}(-\log \cos x) = \tan x$

$\Rightarrow \int \tan x\, dx = -\log|\cos x|$

(xiii) $\frac{d}{dx}(\log(\sec x + \tan x)) = \sec x \Rightarrow$

$\int \sec x dx = \log|\sec x + \tan x|$

(xiv) $\frac{d}{dx}(\log(\cos ec x - \cot x)) = \cos ec x$

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$\Rightarrow \int \cos ec x\, dx = \log|\cos ec x - \cot x|$

(xv) $\frac{d}{dx}\left(\sin^{-1}\frac{x}{a}\right) = \frac{1}{\sqrt{a^2-x^2}} \Rightarrow$

$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right)$

(xvi) $\frac{d}{dx}\left(\cos^{-1}\frac{x}{a}\right) = -\frac{1}{\sqrt{a^2-x^2}}$

$\int -\frac{1}{\sqrt{a^2-x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right)$

(xvii) $\frac{d}{dx}\left(\frac{1}{a}\tan^{-1}\frac{x}{a}\right) = \frac{1}{a^2+x^2}$

$\Rightarrow \int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

(xviii) $\frac{d}{dx}\left(\frac{1}{a}\cot^{-1}\frac{x}{a}\right) = -\frac{1}{a^2+x^2}$

$\Rightarrow \int -\frac{1}{a^2+x^2} dx = \frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right) + C$

(xix) $\frac{d}{dx}\left(\frac{1}{a}\sec^{-1}\frac{x}{a}\right) = \frac{1}{x\sqrt{x^2-a^2}}$

$\Rightarrow \int \frac{1}{x\sqrt{x^2-a^2}} dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$

(xx) $\frac{d}{dx}\left(\frac{1}{a}\cos ec^{-1}\frac{x}{a}\right) = -\frac{1}{x\sqrt{x^2-a^2}}$

$\Rightarrow \int -\frac{1}{x\sqrt{x^2-a^2}} dx = \frac{1}{a}\cos ec^{-1}\left(\frac{x}{a}\right) + C$

Some Important Integrations

(i) $\int (ax+b)^n dx = \frac{1}{a}\frac{(ax+b)^{n+1}}{n+1} + C, n \neq -1$

(ii) $\int \frac{1}{ax+b} dx = \frac{1}{a}\log|ax+b| + C$

(iii) $\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + C$

(iv) $\int a^{bx+c} dx = \frac{1}{b}.\frac{a^{bx+c}}{\log a} + C,\ a > 0$ and $a \neq 1$

(v) $\int \sin(ax+b) dx = -\frac{1}{a}\cos(ax+b) + C$

(vi) $\int \cos(ax+b) dx = \frac{1}{a}\sin(ax+b) + C$

(vii) $\int \sec^2(ax+b) dx = \frac{1}{a}\tan(ax+b) + C$

(viii) $\int \cos ec^2(ax+b) dx = -\frac{1}{a}\cot(ax+b) + C$

(ix) $\int \sec(ax+b)\tan(ax+b) dx = \frac{1}{a}\sec(ax+b) + C$

(x) $\int \cos ec(ax+b)\cot(ax+b) dx = -\frac{1}{a}\cos ec(ax+b) + C$

(xi) $\int \tan(ax+b) dx = -\frac{1}{a}\log|\cos(ax+b)| + C$

(xii) $\int \cot(ax+b) dx = \frac{1}{a}\log|\sin(ax+b)| + C$

(xiii) $\int \sec(ax+b) dx = \frac{1}{a}\log|\sec(ax+b) + \tan(ax+b)| + C$

(xiv) $\int \cos ec(ax+b) dx = \frac{1}{a}\log|\cos ec(ax+b) - \cot(ax+b)| + C$

Some Special Integrals

(i) $\int \frac{1}{x^2+a^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

(ii) $\int \frac{1}{x^2-a^2} dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$

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(iii) $\int \frac{1}{a^2-x^2}dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + C$

(iv) $\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + C$

(v) $\int \frac{1}{\sqrt{a^2+x^2}}dx = \log\left|x+\sqrt{a^2+x^2}\right| + C$

(vi) $\int \frac{1}{\sqrt{x^2-a^2}}dx = \log\left|x+\sqrt{x^2-a^2}\right| + C$

Some Important Integrals

(i) $\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{1}{2}a^2\sin^{-1}\left(\frac{x}{a}\right) + C$

(ii) $\int \sqrt{a^2+x^2}\,dx = \frac{1}{2}x\sqrt{a^2+x^2} + \frac{1}{2}a^2\log\left|x+\sqrt{a^2+x^2}\right| + C$

$\int \sqrt{x^2-a^2}\,dx = \frac{1}{2}x\sqrt{x^2-a^2} - \frac{1}{2}a^2\log\left|x+\sqrt{x^2-a^2}\right| + C$

Important Note: To evaluate integrals of the form $\int \sin mx\cos nx dx$, $\int \sin mx \sin nx\, dx$, $\int \cos mx \cos nx\, dx$ and $\int \cos mx \sin nx\, dx$, we use the following trigonometrical identities

Identities:

$2\sin A\cos B = \sin(A+B)+\sin(A-B)$:

$2\cos A\sin B = \sin(A+B)-\sin(A-B)$

$2\cos A\cos B = \cos(A+B)+\cos(A-B)$:

$2\sin A\sin B = \cos(A-B)-\cos(A+B)$

Integrals Of The Form $\int \sin^m x\cos^n x dx$, Where *m*, *n* are Positive Integers

In the integrals of the form $\int \sin^m x\cos^n x dx$ the following substitutions are useful.

(i) If *m* is odd i.e., power of $\sin x$ is odd, put $\cos x = t$

(ii) If *n* is odd i.e., power of $\cos x$ is odd, put $\sin x = t$

(iii) If both *m* and *n* are even, then use De'Moivre's theorem.

Some Important Substitutions — Following are some substitutions useful in evaluating integrals.

Expression	Substitution
a^2+x^2	$x = a\tan\theta$ or $a\cot\theta$
a^2-x^2	$x = a\sin\theta$ or $a\cos\theta$
x^2-a^2	$x = a\sec\theta$ or $a\,\mathrm{cosec}\,\theta$
$\sqrt{\frac{a-x}{a+x}}$ or $\sqrt{\frac{a+x}{a-x}}$	$x = a\cos 2\theta$

Integrals Of The Type $\int \frac{1}{ax^2+bx+c}dx$

To evaluate this of integrals we express ax^2+bx+c as the sum or difference of two squares by using the following steps.

STEP I Make the coefficient of x^2 unity by taking it common

STEP II Add and subtract the square of half of the coefficient of x

Integrals Of The Type $\int \frac{1}{\sqrt{ax^2+bx+c}}dx$

To evaluate this type of integrals, we express ax^2+bx+c as the sum or difference of two squares by using the following steps.

STEP I Make the coefficient of x^2 unity by taking it common.

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Integrals Of The Form $\int \frac{px+q}{ax^2+bx+c}$

To evaluate this type of integrals we express the numerator as follows:

$px+q = \lambda$ (Diff. of denominator) $+\mu$ $= \lambda(2ax+b)+\mu$

Integrals Of The Form $\int \frac{P(x)}{ax^2+bx+c}dx$

Where P(X) is Polynomial of Degree Greater Than or Equal To 2

To evaluate this type of integrals we divide the numerator by the denominator and express the integrand as

$$Q(x) + \frac{R(x)}{ax^2+bx+c}$$

where R(x) is a linear function of x.

$\therefore \int \frac{P(x)}{ax^2+bx+c}dx = \int Q(x)dx + \int \frac{R(x)}{ax^2+bx+c}dx$

Integrals Of The Form $\int \frac{px+q}{\sqrt{ax^2+bx+c}}dx$

To evaluate this type of integrals we express the numerator as follows

$px+q = \lambda$ (Diff. of denominator) $+\mu = \lambda(2ax+b)+\mu$

where λ and μ are constants to be determined by equating the coefficients of similar terms on both sides. So we have

Integrals Of The Form $\int \frac{1}{a\sin^2 x+b\cos^2 x}dx$,

$\int \frac{1}{a+b\sin^2 x}dx$, $\int \frac{1}{a+b\cos^2 x}dx$, $\int \frac{1}{(a\sin x+b\cos x)^2}dx$,

$\int \frac{1}{a+b\sin^2 x+c\cos^2 x}$

To evaluate this type of integrals we procecd as follows

STEP I Divide numerator and denominator both by $\cos^2 x$

STEP II Replace $\sec^2 x$, if any, in denominator by $1+\tan^2 x$

STEP III Put $\tan x = t$ so that $\sec^2 x dx = dt$

Integrals Of The Form $\int \frac{1}{a\sin x+b\cos x}dx$,

$\int \frac{1}{a+b\sin x}dx$, $\int \frac{1}{a+b\cos x}dx$, $\int \frac{1}{a\sin x+b\cos x+c}dx$

To evaluate this type of integrals we proceed as follows.

STEP 1 Put $\sin x = \frac{2\tan x/2}{1+\tan^2 x/2}$, $\cos x = \frac{1-\tan^2 x/2}{1+\tan^2 x/2}$

STEP 2 Replace $1+\tan^2\frac{x}{2}$ in the numerator by $\sec^2\frac{x}{2}$

STEP 3 Put $\tan\frac{x}{2} = t$ so that $\frac{1}{2}\sec^2\frac{x}{2}dx = dt$

Integrals Of The Form $\int \frac{a\sin x+b\cos x}{c\sin x+d\cos x}dx$

To evaluate this type of integrals we express the numerator as follows.

Numerator = λ(Diff. of denominator) + μ (denominator)

i.e. $(a\sin x+b\cos x) = \lambda..\frac{d}{dx}(c\sin x+d\cos x) + \mu(c\sin x+d\cos x)$

where λ and μ are constants to be determined by comparing the coefficents of $\sin x$ and $\cos x$ on both sides.

$$\therefore \int \frac{a\sin x+b\cos x}{c\sin x+d\cos x}dx$$

$$= \int \frac{\lambda(c\cos x-d\sin x)+\mu(c\sin x+d\cos x)}{c\sin x+d\cos x}dx$$

$$= \int \mu\, dx + \lambda\int \frac{c\cos x-d\sin x}{c\sin x+d\cos x}dx \Rightarrow \;= \mu x+\lambda\log|c\sin x+d\cos x| + K$$

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Integrals Of The Form $\int \frac{a\sin x + b\cos x + c}{p\sin x + q\cos x + r}dx$

To evaluate this type of integrals, we express the numerator as follows

Numerator = λ (denominator) + μ (Diff. of denominator) + V i.e.,

$(a\sin x + b\cos x + c) = \lambda\ (p\sin x + q\cos x + r)$
$+\mu(p\cos x - q\sin x) + v$

Where λ, μ, v are constants to be determined by comapring the coefficients of $\sin x, \cos x$ and constant term on both sides.

$$\therefore \int \frac{a\sin x + b\cos x + c}{p\sin x + q\cos x + r}dx$$

$$= \int \lambda\, dx + \mu \int \frac{\text{Diff. of denominator}}{\text{denominator}}dx$$

$$+ v\int \frac{1}{p\sin x + q\cos x + r}\, dx$$

$$= \lambda x + \mu\ \log|\text{denominator}| + v\int \frac{1}{p\sin x + q\cos x + r}dx$$

INTEGRATION BY PARTS

Theorem: If *u* and *v* are two functions of *x*, then

$$\int uv dx = u\left(\int v dx\right) - \int\left\{\frac{du}{dx}\int v dx\right\}dx$$

i.e. the integral of the product of two functions = (First function) × (Integral of second function) – integral of {(Diff. of first function) ×(integral of second function)}

<u>Note 1</u> Proper choice of first and second function

Integration with the help of the above rule is called the integration by parts. In the above rule there are two terms on RHS and in both the terms the integral of the second function is involved. Therefore in the product of two functions if one of the two functions is not directly integrable. (e.g., log *x*, $\sin^{-1}x$, $\tan^{-1}x$ etc.) we take it as the first function and the remaining function is taken as the second function. If there is no other function, then unity is taken as the second function. If in the integral both the functions are easily integrable, then the first function is chosen in such a way that the derivative of the function is a simple function and the function thus obtained under the integral sign is easily integrable than the original function.

<u>Note 2</u> We can also choose the first function as the function which comes first in the word **ILATE**, where

I – Stands for the inverse trigonometric function ($\sin^{-1}x, \cos^{-1}x, \tan^{-1}x$ etc.)

L – Stands for the logarithmic functions

A – Stands for the algebraic functions.

T – Stands for the trigonometric functions.

E – Stands for the exponential functions

Integrals of The Form $\int e^x\{f(x) + f'(x)\}dx =$

$$= \int \underset{II}{e^x} \underset{I}{f(x)}\, dx + \int e^x f'(x)dx$$

$$= f(x).e^x - \int f'(x)e^x dx + \int e^x f'(x)dx + C = e^x f(x) + C$$

Integrals Of The Form $\int (px+q)\sqrt{ax^2+bx+c}dx$

To evaluate integrals of the type $\int (px+q)\sqrt{ax^2+bx+c}dx$, we express the linear factor $px+q$ as follows

$$px + q = \lambda.\frac{d}{dx}(ax^2+bx+c) + \mu$$

Integrals of The Form $\int \frac{x^2+1}{x^4+\lambda x^2+1}dx$, $\int \frac{x^2-1}{x^4+\lambda x^2+1}dx, \int \frac{1}{x^4+\lambda x^2+1}\, dx$

Where λ is a Constant

To evaluate this type of integrals, divide the numerator and denominator by x^2 and put $x + \frac{1}{x} = t$ or $x - \frac{1}{x} = t$, which ever or differentiation gives the numerator of the resulting integrand.

Integration Of Some Special Irrational Algebraic Functions

In this article we shall discuss four integrals of the form $\int \frac{\phi(x)}{P\sqrt{Q}}dx$, where P and Q are polynomial functions of *x*.

Integrals of the form $\int \frac{\phi(x)}{P\sqrt{Q}}dx$**, where p and q both are linear functions of** ***x***. To evaluate this type of integrals we put $Q = t^2$ i.e. to evaluate integrals of the form $\int \frac{1}{(ax+b)\sqrt{cx+d}}dx$ put $cx + d = t^2$

Integrals Of The Form $\int \frac{\phi(x)}{P\sqrt{Q}}dx$**, Where P Is A Quadratic Expression And Q Is A Linear Expression** —To evaluate this type of integrals we put $Q = t^2$ i.e, to evaluate integrals of the form $\int \frac{1}{(ax^2+bx+c)\sqrt{px+q}}dx$, put $px + q = t^2$

Integrals Of The Form $\int \frac{\phi(x)}{P\sqrt{Q}}dx$**, Where P Is A Linear Expression And Q Is A Quadratic Expression** —To evaluate this type of integrals we put $p = 1/t$ i.e to evaluate integrals of the form $\int \frac{1}{(ax+b)\sqrt{px^2+qx+r}}dx$, put $ax + b = \frac{1}{t}$

Integrals Of The Form $\int \frac{\phi(x)}{P\sqrt{Q}}dx$**, Where P And Q Both Are Pure Quadratic Expression In —** x i.e. $P = ax^2 + b$ and $Q = cx^2 + d$ To evaluate this type of integrals we put $x = \frac{1}{t}$ and then $c + dt^2 = u^2$ i.e. to evaluate integrals of the form $\int \frac{1}{(ax^2+b)\sqrt{cx^2+d}}dx$, we put $x = \frac{1}{t}$ to obtain $\int \frac{-tdt}{(a+bt^2)\sqrt{c+dt^2}}$ and then $c + dt^2 = u^2$

S.No.	Form of the rational function	Form of the partial fraction
1.	$\frac{px+q}{(x-a)(x-b)}$, $a \neq b$	$\frac{A}{x-a} + \frac{B}{x-b}$
2.	$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2}$
3.	$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$
4.	$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$
5.	$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$,
	where $x^2 + bx + c$ cannot be factorised further	

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